р	is the pressure;
Т	is the temperature;
h	is the enthalpy;
v	is the specific volume;
Pr	is the Prandtl number;
Z	is the dryness level;
ĸ	is the adiabatic exponent of the heated vapor;
•	

 λ, μ, ν are the coefficients of thermal conductivity, dynamic and kinematic viscosity, respectively.

The subscripts l and v refer, respectively, to parameters of the liquid and vapor phase; e refers to parameters of the external flow.

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DISCHARGE AND DYNAMIC CHARACTERISTICS OF BOILING

WATER IN LAVAL NOZZLES

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The criterial processing of experimental results, obtained by studying the discharge of hot water into the atmosphere through a Laval nozzle, is proposed. The initial parameters of the water were varied over the range $p_0 = 4.9-17.6$ bar and $t_0 = 119-204^{\circ}C$.

The effect is investigated in this paper of the initial parameters of hot water, p_0 and t_0 [$t_0 > t_s(p_n)$], where $t_s(p_n)$ is the saturation temperature at the temperature of the surrounding medium, on the discharge characteristics, the mean-mass efflux velocities, and pressures for nozzles of specified geometry.

The experiments were conducted in a facility representing a hydraulic circuit, including a high-pressure tank, the working section — the nozzle, and a catcher vessel. In order to produce water with specified initial parameters, a number of anciliary devices were provided: a heat-exchanger; electric heater; a compressed air supply line; and pumps.

The experiments were carried out with water that had not undergone preliminary deaeration and purification; however, during heating up in the closed volume, a periodic pressure "scouring" was carried out, which led to a reduction of the amount of gas in the water. The water obtained was underheated by 1-2° before the saturation line. Greater underheatings were achieved by the addition of compressed air.

During the experiment, the initial temperature of the water (t_0) , the initial pressure (p_0) , the static pressure along the nozzle (p_i) , the water flow rate per second (G), and the reactive thrust (R) were measured. From the measured values of G, R, and p_0 , the mean flow velocity at the outlet was calculated by the formula

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Fig. 1. Dependence of critical cavitation number $\sigma^* = [\mathbf{p}_{\mathbf{S}}(t_0) - \mathbf{p}_{th}^*]/[\mathbf{p}_0 - \mathbf{p}_{th}^*]$ on the initial dimensionless subheating of the liquid $1 - \mathbf{p}_{\mathbf{S}}(t_0)/\mathbf{p}_0$; 1) authors' experimental data; 2) [1]; 3) [2]; 4) [3]; 5) [4]; 6) [5] - water; 7) [5] Freon-113.

$$w_a \coloneqq [R - (p_0 - p_n) F_a]/G$$

Tests were conducted on two Laval nozzles, representing channels formed by two cones. The dimensions of the flow section of nozzle No. 1 were calculated by the model of equilibrium adiabatic-homogeneous flow for the initial conditions: $p_0 = 4.9$ bar, $t_0 = 144^{\circ}$ C, $p_n = 1$ bar, and G = 1 kg/sec. The dimensions of nozzle No. 2 were set by the condition of similarity of the expanding parts of the channel. After preparation, the measurements were carried out and the following data obtained for nozzle No. 1: $d_{th} = 10.0$ mm, $d_a = 34.6$ mm, length of the divergent section l = 394 mm, angle of convergent section $2\beta = 16^{\circ}30^{\circ}$, angle of divergent section $2\alpha = 3^{\circ}35^{\circ}$; for nozzle No. 2, $d_{th} = 5.0$ mm, $d_a = 18.6$ mm, l = 200 mm, $2\beta = 21^{\circ}20^{\circ}$, and $2\alpha = 3^{\circ}54^{\circ}$. In nozzle No. 2, the throat section was constructed in the form of a cylindrical insert with a length of 7 mm. The nozzle was drained lengthwise.

The experimental investigations were conducted over the range of variations $p_0 = 4.9-17.6$ bar, $t_0 = 119-204^{\circ}C$, and $p_n \ge 1$ bar.

The flow characteristics of a Laval nozzle are determined by the processes in the throat, and the flowrate per second G, for the efflux of water subheated up to the saturation line, can be determined by Bernouilli's equation

$$G = F_{\rm th} \sqrt{2\rho_0 (p_0 - p_{\rm th})}.$$
 (1)

The central moment of the problem being considered is the determination of the value of p_{th} (the suffix th denotes throat).

The basic assumption for the proposed processing of the experiments is the representation of the flow in the nozzle throat as the motion of a cavitating liquid, defined by the cavitation number σ :

$$\sigma = [p_s(t_0) - p_{th}] [0.5\rho_0 w_{th}^2 = [p_s(t_0) - p_{th}] [p_0 - p_{th}].$$

This assumption is based on the results of a study of the process of origination and development of the vapor phase during the flow of hot water, discussed in detail in [6].

Even in the presence of a small pressure drop $\Delta p = p_0 - p_n$, in view of the dynamic reduction of pressure in the throat, the conditions are created for the formation of a cavitation void. When the pressure is established close to $p_s(t_0)$, a "vapor cloudlet" forms, which rapidly collapses downstream. With further reduction of pressure, the vapor region expands, filling an even larger and larger part of the nozzle. When the relative counterpressure $\varepsilon_1^* = p_n/p_0$ reaches a value at which a change of the external pressure ceases to affect the parameters in the throat, the flow rate attains the maximum value.



Fig. 2. Criterial processing of the experimental pressure values in different sections of nozzles No. 1 (black points) and No. 2 (open points): a) $F_i/F_{th} = 3.5$; b) 6.2; c) 9.0; d) 11.8; e) 13.9; 1) $p_0 = 5.9$ bar; 2) 7.8; 3) 9.8; 4) 11.7; 5) 13.7; 6) 17.6.

It has been experimentally established [6] that with an initial dimensionless subheating of the water $[1 - p_s(t_0)/p_0] \le 0.7$, the value $\varepsilon_1^* > 0.2$, i.e., in the range of initial values of p_0 and t_0 investigated, with efflux into the atmosphere, the nozzle operates in conditions of a supercritical pressure drop. In these conditions, the problem reduces to the determination of the function

$$\sigma^* = f \left[1 - p_s(t_0) / p_0 \right].$$

Figure 1 shows the dependence of the cavitation number in the conditions of the critical flow cycle σ^* , on the initial dimensionless subheating of the liquid $1 - p_s(t_0)/p_0$; the values of σ^* have been calculated from the experimental results obtained both by us in this investigation and by other authors [1-5]. The data on the discharge of Freon-113 [5], processed in the proposed coordinates, are also shown in Fig. 1.

Comparison of the experimentally measured flow rates with the values calculated by Eq. (1), if p_{th}^* is determined by Fig. 1, gives completely satisfactory agreement.

In the expanding part of the nozzle, as analysis of the experimental data shows, the discharge process of the subheated liquid is determined by the initial parameters $(p_0, t_0, and p_n)$, the thermophysical properties of the liquid (c_p, r) , and the geometrical dimensions of the channel $(d_{th}, d_a, and l)$.

The dimensional analysis carried out leads to the conclusion that, in the problem being considered, the required functions have the form

$$p_{i}/p_{s}(t_{n}) = f_{1}(K, P_{n}/P_{0s} | l_{i}/d_{th} | d_{i}/d_{th}),$$

$$w_{a}/\sqrt{c_{p}[t_{0}-t_{s}(p_{n})]} = f_{2}(K, p_{n}/p_{0}, l_{a}/d_{th}, d_{a}/d_{n}),$$

where l_i is the distance from the nozzle throat to the *i*-th section.

Figures 2 and 3 show the criterial processing of the experimental pressure values in certain comparative sections of nozzles Nos. 1 and 2, and the discharge velocity for these nozzles. The results are satisfactorily generalized by the relations

$$p_i/p_s(t_0) = f'(K \cdot p_n/p_0), \quad w_a/\sqrt{c_p[t_0 - t_s(p_n)]} = f''(K \cdot p_n/p_0)$$

The dispersity between the discharge velocity curves for nozzles 1 and 2 obviously is due to inaccurate similarity: the deviation in the outlet areas amounts to 15%.

It has not been possible to obtain a generalized dependence of the nozzle velocity coefficient φ on the performance parameters. The quantity φ can be calculated easily from the discharge velocity w_q



Fig. 3. Criterial processing of the experimental values of the discharge meanmass velocity A = $(w_a / \sqrt{c_p[t_0 - t_s(p_n)]})$; a) nozzle No. 1, b) No. 2; 1) $p_0 = 5.9$ bar; 2) 7.8; 3) 9.8; 4) 11.7; 5) 13.7; 6) 17.6. Open points) nozzle No. 1, black points) nozzle No. 2.

$q = w_{a} w_{max},$

where w_{max} is the discharge velocity with isentropic expansion up to the pressure of the surrounding medium.

NOTATION

р	is the pressure;
t	is the temperature;
$\Delta t = t_s(p_0) - t_0$	is the initial subheating;
w	is the mean-mass velocity;
c _p	is the specific heat of liquid;
ρ ^F	is the density of liquid;
r	is the heat of vaporization;
R	is the reactive thrust;
F	is the area;
G	is the flow rate per sec;
1	is the length;
d	is the diameter;
$K = c_p \Delta t/r$	is the phase conversion criterion.
-	

Subscripts

- 0 is the initial parameters;
- *a* is the outlet section of nozzle;
- i is the i-th section of nozzle;
- th is the throat section;
- n is the surrounding medium;
- s is the saturation line;
- * is the critical conditions.

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TURBULENT FORCED FLOW AND HEAT EXCHANGE IN VERTICAL CHANNELS IN CONDITIONS OF FREE CONVECTION

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Based on a unified approach, data are analyzed and generalized concerning the distributions of velocity and temperature, frictional resistance and heat transfer in the case of turbulent free-convective flow, and forced flow in conditions of the significant effect of the gravitational field.

Turbulent forced flow in vertical channels from below upwards with heating, and from above downwards with cooling, is considered under the conditions of action of the gravitational field. We call the limiting case of the very strong effect of buoyancy, the "free convection mode."

First of all, we consider free turbulent convection close to the vertical surfaces from the general positions of the boundary flows.

In the first place, we will be interested in a boundary condition of the second species, and therefore we describe the dimensionless numbers related with the thermal effect by the quantity q_c . However, heat exchange in the case of turbulent forced flow is almost no different with the boundary conditions $q_c = \text{const}$ and $t_c = \text{const}$. Even the turbulent free convective flow is quite conservative during transition from the boundary condition $q_c = \text{const}$ to $t_c = \text{const}$. In particular, the data of a number of papers confirm this, showing the independence of the heat-transfer coefficient $\alpha = q_c/(t_c - t_{\infty})$ on the longitudinal coordinate x, i.e., for a specified constant value of q_c or t_c the other quantity correspondingly is also constant.

In Fig. 1 the temperature distribution in the case of forced turbulent boundary flow of air [1, 2] and turbulent free convective flow of air along a vertical plate [3] are compared in the universal coordinates $T^+ - \eta$. In [3] the tangential stress on the wall τ_c was measured so that the friction velocity v_* is determined by the experimental data. It can be seen that the temperature distribution in universal coordinates in the case of free convection coincides with the temperature distribution in the case of forced flow without the effect of mass forces. This distribution is described by the following interpolation relation:

$$T_{\rm r}^{+} = 2.2\ln\left(1 + 0.45\Pr\eta\right) + (13\Pr^{2/3} - \ln\Pr - 4)\left[1 - \exp\left(-\Pr^{3/4}\eta^{1.5}/50\right)\right],\tag{1}$$

corresponding with an accuracy of $\pm 7\%$ to the most reliable experimental data assembled in [1, 2, 4, 5], and the results of calculations given in these papers, over the range of Pr values from 0.02 to 64.

In [6], for $Pr \cong 16$, the relation

$$\frac{\tau_c}{\rho \left[\beta g \left(t_c - t_{\infty}\right) v\right]^{2/3}} = \text{const}$$
⁽²⁾

is obtained, from which it follows that

$$\operatorname{Gr}_{x} T^{+}_{\infty} / x^{4}_{+} \operatorname{Pr} = A = 0.18,$$
 (2a)

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